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THE APPLICATION OF THERMAL AND CREEP EFFECTS TO THE COMBINED ISOTROPIC-KINEMATIC HARDENING MODEL FOR INELASTIC STRUCTURAL ANALYSIS BY THE FINITE ELEMENT METHOD

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David H. Allen* and Walter E. Haisler**

ABSTRACT

In this paper, a formulation is presented for a combined isotropic-kinematic hardening rule which includes both creep and thermal effects. The resulting model accounts for temperature dependent material properties as well as the Bauschinger effect for reverse loading. The virtual work equation and resulting matrix equations are derived for implementation of the model into a general purpose finite element code.

INTRODUCTION

In the past decade considerable research requiring structural analysis including thermal and creep effects has been done in fields such as laser technology and nuclear reactor analysis. Thus, it is desirable to modify existing nonlinear structural analysis codes to encompass thermal and creep effects.

Researchers at the Massachusetts Institute of Technology[1] as well as the Oak Ridge National Laboratory[2] have included thermal and creep effects in structural analysis programs for the cases of isotropic hardening and kinematic hardening applied to metal plasticity. However, as has been pointed

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out by Haisler[3], there may be certain material types and stress histories for which these models are inadequate.

One model which has been shown to predict cyclic loadings quite well for metals under isothermal conditions is the combined isotropic-kinematic hardening rule[4]. This paper outlines the formulation used to obtain the finite element equations necessary to include thermal and creep effects in a combined isotropic-kinematic hardening model currently in the general purpose code AGGIE I[5].

In the first section of this paper, the combined isotropic-kinematic workhardening rule will be utilized to obtain a material model which accounts for creep and thermal effects. Next, the derivation of the linear virtual work equation within a total Lagrangian description is presented for the material model described above. The constitutive law will then be applied to the virtual work expression and the resulting relation will be discretized to obtain a matrix formulation which is applicable to finite element modeling. The iterative technique necessary to obtain equilibrium for the nonlinear virtual work equation will then be presented. Finally, the constitutive law will be specialized for certain load conditions and the resulting matrix equations will be given.

THE CONSTITUTIVE RELATION

In order to determine the quantities in the constitutive law, it is necessary to employ an appropriate material modeling theory. The model used in this report is the incremental theory of plasticity. In this method a yield criterion such as the Von Mises yield condition is used with an associated flow rule and a so-called workhardening rule.

The workhardening rule utilized here is the combined isotropic-kinematic hardening rule, which accounts for both expansion and translation of the yield surface in stress space and is described by the yield function

$$F(S_{ij} - \alpha_{ij}) = k^2(\int dE^P_{ij}, T) \tag{1}$$

where

α_{ij} = coordinates of the yield surface center in stress space,

dE^P_{ij} = plastic strain increment tensor,

T = temperature,

and $\int dE^P_{ij}$ represents the plastic strain history dependence in the yield function.

The absence of temperature dependence in α_{ij} on the left-hand side of the above statement signifies that there is assumed to be no translation of the yield surface due to temperature changes. Hence, thermally induced yield surface modifications are assumed to be strictly isotropic. Phillips[10] has shown in his experimental work involving metals that although thermally induced yield surface modifications may be slightly anisotropic for highly distorted yield surfaces, the absence of translation is verified for any yield surface in stress space. An identical assumption is used in the kinematic hardening rules employed at the Oak Ridge National Laboratory and the Massachussets Institute of Technology.

From the above yield function it can be seen that a statement of consistency during plastic loading is given by

$$\frac{\partial F}{\partial S_{ij}}\, dS_{ij} - \frac{\partial F}{\partial S_{ij}}\, d\alpha_{ij} - 2k dk\,(\int dE^P_{ij}) - 2k\frac{\partial k}{\partial T}\, dT = 0 \tag{2}$$

where $dk(\int dE^P_{ij})$ is defined to be the change in the yield surface size with respect to the history of plastic strain. Because the above consistency condition is difficult to employ directly, the consistent statement employed by Ziegler for isothermal kinematic hardening is modified and utilized here. The applicability of the Ziegler modification to combined isotropic-kinematic hardening has been previously established by this author[7]. As employed in

the combined isotropic-kinematic hardening rule the Ziegler modification is

$$\frac{\partial F}{\partial S_{ij}} \, c dE^{P}_{ij} = \frac{\partial F}{\partial S_{ij}} \, d\alpha_{ij} + 2kdk \, (\int dE^{P}_{ij}) \tag{3}$$

where c is a scalar constant (called the hardening modulus) to be determined.

Substituting the above relation into equation (2) gives

$$\frac{\partial F}{\partial S_{ij}} \, dS_{ij} - \frac{\partial F}{\partial S_{ij}} \, c dE^{P}_{ij} - 2k\frac{\partial k}{\partial T} \, dT = 0 \tag{4}$$

The above consistency condition has the added advantage that it can be interpreted as a statement controlling nonisothermal loading and unloading. Since during neutral loading the plastic strain increment is zero, a statement representing plastic loading would be

$$\frac{\partial F}{\partial S_{ij}} \, dS_{ij} - 2k\frac{\partial k}{\partial T} \, dT > 0 \tag{5}$$

and it follows that a statement governing unloading is

$$\frac{\partial F}{\partial S_{ij}} \, dS_{ij} - 2k\frac{\partial k}{\partial T} \, dT < 0 \tag{6}$$

It can be seen that thermally induced changes in the yield surface size can be significant in affecting the loading process. For example, equation (5) correctly predicts loading when the stress increment is zero and the temperature increment is positive since $\partial k/\partial T$ is negative for most metals. In fact, a small negative stress increment may even produce loading if the temperature increment produces a large enough contraction of the yield surface to ensure that equation (5) remains positive.

The total stress at time t for a material with no strain rate dependence is

$$S^{t}_{ij} = D^{t}_{ijmn} \, (E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}) \tag{7}$$

where

D^{t}_{ijmn} = elastic modulus tensor,

E^{Ct}_{mn} = creep strain tensor, and

E^{Tt}_{mn} = thermal strain tensor.

The total differential of the above equation is

$$dS_{ij} = D^t_{ijmn}\,(dE_{mn}-dE^P_{mn}-dE^C_{mn}-dE^T_{mn}) + dD_{ijmn}(E^t_{mn}-E^{Pt}_{mn}-E^{Ct}_{mn}-E^{Tt}_{mn}) \tag{8}$$

where the increment dD_{ijmn} is caused by a temperature change.

The above equation is exact only in the limit as the stress increment approaches zero. In spite of this fact, many researchers have used equation (8) to define the incremental constitutive relation for finite load and temperature steps. Unless the material properties vary significantly with temperature, the above approximation is satisfactory, but for the case where material properties depend heavily on temperature (e.g., above half the material melting point in many metals) an additional term should be added to account for finite step sizes. To obtain this term, consider the stress at time $t+\Delta t$, which is given by

$$S^{t+\Delta t}_{ij} = D^{t+\Delta t}_{ijmn}\,(E^{t+\Delta t}_{mn} - E^{Pt+\Delta t}_{mn} - E^{Ct+\Delta t}_{mn} - E^{Tt+\Delta t}_{mn}) \tag{9}$$

The change in stress during a load step is then given by the difference in equations (9) and (7):

$$dS_{ij} = S^{t+\Delta t}_{ij} - S^t_{ij} = D^{t+\Delta t}_{ijmn}\,(E^{t+\Delta t}_{mn} - E^{Pt+\Delta t}_{mn} - E^{Ct+\Delta t}_{mn} - E^{Tt+\Delta t}_{mn})$$
$$- D^t_{ijmn}\,(E^t_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}) \tag{10}$$

Equation (10) may be written equivalently as follows:

$$dS_{ij} = (D^t_{ijmn} + dD_{ijmn})\,(E^t_{mn} + dE_{mn} - E^{Pt}_{mn} - dE^P_{mn}$$
$$- E^{Ct}_{mn} - dE^C_{mn} - E^{Tt}_{mn} - dE^T_{mn}) - D^t_{ijmn}\,(E^t_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}) \tag{11}$$

A further refinement of equation (11) yields

$$dS_{ij} = D^t_{ijmn}\,(dE_{mn} - dE^P_{mn} - dE^C_{mn} - dE^T_{mn})$$
$$+ dD_{ijmn}\,(E^t_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}) + dD_{ijmn}\,(dE_{mn}-dE^P_{mn}-dE^C_{mn}-dE^T_{mn}) \tag{12}$$

A comparison of equations (8) and (12) reveals that the differential form neglects the second order effect produced by the last term in equation (12). Our experience shows that the second order term may be neglected in all cases except one; namely, that the temperature change during a load step produces an elastic constitutive tensor modification which will cause the last term in equation (12) to become significant when compared to the first two terms. Since we have already encountered test cases where this term is significant,we have included it in our formulation. We have chosen for programming purposes, however, to write equation (12) in the following form:

$$dS_{ij} = D^{t+\Delta t}_{ijmn}\,(dE_{mn} - dE^{P}_{mn} - dE^{C}_{mn} - dE^{T}_{mn}) + dD_{ijmn}(E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}) \qquad (13)$$

Now recall that the normality condition requires that

$$dE^{P}_{ij} = d\lambda \frac{\partial F}{\partial S_{ij}} \qquad (14)$$

Substituting this relation into equations (4) and (13) gives

$$\frac{\partial F}{\partial S_{ij}}\,dS_{ij} - \frac{\partial F}{\partial S_{ij}}\,c\,d\lambda\,\frac{\partial F}{\partial S_{ij}} - 2k\frac{\partial k}{\partial T}\,dT = 0 \qquad (15)$$

and

$$dS_{ij} = D^{t+\Delta t}_{ijmn}\,(dE_{mn} - d\lambda\frac{\partial F}{\partial S_{mn}} - dE^{C}_{mn} - dE^{T}_{mn}) + dD_{ijmn}\,(E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}) \qquad (16)$$

Next, substitute equation (16) into equation (15) and solve for $d\lambda$:

$$d\lambda = \Big[\frac{\partial F}{\partial S_{ij}}\,D^{t+\Delta t}_{ijmn}\,dE_{mn} - \frac{\partial F}{\partial S_{ij}}\,D^{t+\Delta t}_{ijmn}\,dE^{C}_{mn} - \frac{\partial F}{\partial S_{ij}}\,D^{t+\Delta t}_{ijmn}\,dE^{T}_{mn} + \frac{\partial F}{\partial S_{ij}}\,dD_{ijmn}\,E^{t}_{mn} - \frac{\partial F}{\partial S_{ij}}\,dD_{ijmn}\,E^{Pt}_{mn} - \frac{\partial F}{\partial S_{ij}}\,dD_{ijmn}\,E^{Ct}_{mn} - \frac{\partial F}{\partial S_{ij}}\,dD_{ijmn}\,E^{Tt}_{mn} - 2k\frac{\partial k}{\partial T}\,dT\Big] \Big/ \Big[c\,\frac{\partial F}{\partial S_{ij}}\,\frac{\partial F}{\partial S_{ij}} + D^{t+\Delta t}_{ijmn}\,\frac{\partial F}{\partial S_{ij}}\,\frac{\partial F}{\partial S_{mn}}\Big] \qquad (17)$$

Now substitute equation (17) back into equation (16):

$$dS_{ij} = D^{t+\Delta t}_{ijmn}\Big[dE_{mn} - \frac{\partial F}{\partial S_{mn}} \Big(\frac{\partial F}{\partial S_{tu}} D^{t+\Delta t}_{tuvw}\, dE_{vw}$$

$$- \frac{\partial F}{\partial S_{tu}} D^{t+\Delta t}_{tuvw}\, dE^{C}_{vw} - \frac{\partial F}{\partial S_{tu}} D^{t+\Delta t}_{tuvw}\, dE^{T}_{vw} + \frac{\partial F}{\partial S_{tu}} dD_{tuvw}\, E^{t}_{vw}$$

$$- \frac{\partial F}{\partial S_{tu}} dD_{tuvw}\, E^{Pt}_{vw} - \frac{\partial F}{\partial S_{tu}} dD_{tuvw}\, E^{Ct}_{vw} - \frac{\partial F}{\partial S_{tu}} dD_{tuvw}\, E^{Tt}_{vw}$$

$$- 2k\frac{\partial k}{\partial T}\, dT \Big) \Big/ \Big(c\, \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}} \Big) - dE^{C}_{mn} - dE^{T}_{mn} \Big]$$

$$+ dD_{ijmn}\, (E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}) \tag{18}$$

Rewrite equation (18) in the following form

$$dS_{ij} = \left(D^{t+\Delta t}_{ijmn} - \frac{D^{t+\Delta t}_{ijvw} \frac{\partial F}{\partial S_{vw}} \frac{\partial F}{\partial S_{tu}} D^{t+\Delta t}_{tumn}}{c\, \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}}} \right) (dE_{mn} - dE^{C}_{mn} - dE^{T}_{mn})$$

$$- \left(\frac{D^{t+\Delta t}_{ijvw} \frac{\partial F}{\partial S_{vw}} \frac{\partial F}{\partial S_{tu}} dD_{tumn}}{c\, \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}}} \right) (E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn})$$

$$+ \left(\frac{D^{t+\Delta t}_{ijmn}\, 2k\frac{\partial k}{\partial T} \frac{\partial F}{\partial S_{mn}}}{c\, \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}}} \right) dT$$

$$+ dD_{ijmn}\, (E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}) \tag{19}$$

or

$$dS_{ij} = C^{t}_{ijmn}\, (dE_{mn} - dE^{C}_{mn} - dE^{T}_{mn}) + dP_{ij} \tag{20}$$

where, noting that the elastic-plastic constitutive tensor is superscripted t because it is obtained by using the hardening modulus at time t,

$$C^{t}_{ijmn} = D^{t+\Delta t}_{ijmn} - \frac{D^{t+\Delta t}_{ijvw} \frac{\partial F}{\partial S_{vw}} \frac{\partial F}{\partial S_{tu}} D^{t+\Delta t}_{tumn}}{c \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}}} \tag{21}$$

and

$$dP_{ij} = -\left(\frac{D^{t+\Delta t}_{ijvw} \frac{\partial F}{\partial S_{vw}} \frac{\partial F}{\partial S_{tu}} dD_{tumn}}{c \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}}} \right) (E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn})$$

$$+ \left(\frac{D^{t+\Delta t}_{ijmn}\, 2k \frac{\partial k}{\partial T} \frac{\partial F}{\partial S_{mn}}}{c \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}}} \right) dT + dD_{ijmn} (E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}) \tag{22}$$

It is of particular interest to note that in the case of isothermal loading, dP_{ij} is zero and the model derived by Hunsaker for isothermal combined isotropic-kinematic hardening is recovered.

The last term in equation (20) arises solely due to the admittance of nonisothermal action. The terms in equation (22) which contain dD_{tumn} represent the stress increment due to a thermally induced change in the elastic constitutive tensor. The term containing $\partial k/\partial T$ in the same equation accounts for the stress increment caused by a modification in the yield surface due to a temperature change.

In order to better understand the term dP_{ij} in the constitutive relation, consider the case where a member is loaded such that the total elastic strain tensor is non-zero and the total plastic strain tensor is zero. Now, if a load increment accompanied by a temperature change is introduced such that the elastic strain increment is zero, a stress increment will occur which is equivalent to the product of the change in the elastic constitutive tensor and the total elastic strain tensor.

In order to completely define the above constitutive relation it is necessary to account for $\partial k/\partial T$, c, and μ. To obtain the first quantity

note that $\partial k/\partial T$ is the yield surface gradient with respect to temperature. This value may be obtained by a suitable interpolation of input uniaxial stress-strain data.

To evaluate c, first solve equation (15) for c. The result is

$$c = \frac{\frac{\partial F}{\partial S_{ij}} \, dS_{ij}}{d\lambda \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}}} - \frac{2k\frac{\partial k}{\partial T} \, dT}{d\lambda \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}}} \tag{23}$$

which may be rewritten

$$c = \frac{\frac{\partial F}{\partial S_{ij}} \, dS_{ij}}{d\lambda \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}}} \left(1 - \frac{2k\frac{\partial k}{\partial T} \, dT}{\frac{\partial F}{\partial S_{ij}} \, dS_{ij}} \right) \tag{24}$$

Now define

$$\frac{2}{3} H' = \frac{\frac{\partial F}{\partial S_{ij}} \, dS_{ij}}{d\lambda \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}}} \tag{25}$$

so that equation (24) may be written

$$c = \frac{2}{3} H' \left(1 - \frac{2k\frac{\partial k}{\partial T} \, dT}{\frac{\partial F}{\partial S_{ij}} \, dS_{ij}} \right) \tag{26}$$

Since H' is a stress invariant it may be determined from uniaxial stress-strain data[9].

The last term in the above equation causes some computational difficulty since it involves the stress increment tensor which is itself a function of the hardening modulus c. This shortcoming has been circumvented by some researchers by simply neglecting it. This assumption is a clear violation of the consistency condition and can cause departures of the stress tensor from the yield surface during loading. Therefore, the thermal dependence of the hardening modulus is not neglected herein.

In order to satisfy the condition that c must always be a positive quantity for a workhardening material (i.e., H' is strictly positive), it is necessary to show that the last term in equation (26) is always less than unity. This may be verified quite simply by rearranging the loading condition given in equation (5).

Finally, to account for μ, recall Ziegler's rule,

$$d\alpha_{ij} = \mu(S_{ij} - \alpha_{ij}) \tag{27}$$

Substitute equation (27) into equation (2) and solve for μ.

$$\mu = \frac{\frac{\partial F}{\partial S_{ij}} dS_{ij} - 2k\frac{\partial k}{\partial T} dT - 2kdk\ (\int dE^{P}_{ij})}{(S_{mn} - \alpha_{mn}) \frac{\partial F}{\partial S_{mn}}} \tag{28}$$

The thermal term in the above equation can be seen to be the correction in the yield surface center translation tensor which accounts for isotropic expansion due to a temperature increment.

This completes the formulation of the constitutive law.

THE INCREMENTAL VIRTUAL WORK EQUATION

In order to formulate a statement of equilibrium which will include thermal effects, first consider the virtual work expression within a total Lagrangian description as previously described by Hunsaker[6]:

$$\int_{V_o} S_{ij}^{t+\Delta t}\ \delta E_{ij}^{t+\Delta t}\ dV = \int_{A} T_k \delta u_k dA + \int_{V_o} \rho_o F_k \delta u_k\ dV \tag{29}$$

where

$S_{ij}^{t+\Delta t}$ = the 2nd Piola-Kirchhoff stress tensor at time $t+\Delta t$ referred to the initial configuration V_o at time $t = 0$,

$\delta E_{ij}^{t+\Delta t}$ = the variation in the Green-Lagrange strains at $t+\Delta t$ referred to the initial configuration V_o,

T_k = the surface tractions at time $t+\Delta t$ referred to the surface of the configuration A,

δu_k = the variation in the displacements,

ρ_o = the local density in the initial configuration, and

F_k = the body force per unit mass at time $t+\Delta t$ referred to the initial configuration V_o.

Since the inclusion of thermal effects into the system will affect only the left hand side of equation (29), make the following simplifying definition:

$$\delta W^{t+\Delta t} = \int_A T_k \delta u_k dA + \int_{V_o} \rho_o F_k \delta u_k \, dV \tag{30}$$

As outlined by Hunsaker, in order to formulate an incremental stiffness procedure, first let

$$S_{ij}^{t+\Delta t} = S_{ij}^{t} + \Delta S_{ij} \tag{31}$$

$$E_{ij}^{t+\Delta t} = E_{ij}^{t} + \Delta E_{ij} \tag{32}$$

where S_{ij}^t and E_{ij}^t are respectively the 2nd Piola-Kirchhoff stress tensor and Green-Lagrange strain tensor at time t referred to the initial configuration, and ΔS_{ij} and ΔE_{ij} are stress and strain increments.

The virtual work expression [equations (29) & (30)] thus becomes

$$\int_{V_o} (S_{ij}^t + \Delta S_{ij}) \, \delta \, (E_{ij}^t + \Delta E_{ij}) \, dV = \delta W^{t+\Delta t} \tag{33}$$

Since the strains at time t are known they need not be varied and equation (33) reduces to

$$\int_{V_o} (S_{ij}^t + \Delta S_{ij}) \, \delta \, (\Delta E_{ij}) \, dV = \delta W^{t+\Delta t} \tag{34}$$

Now decompose the strain increment into linear and nonlinear components:

$$\Delta E_{ij} = \Delta E_{ij}^{L} + \Delta E_{ij}^{NL} \tag{35}$$

where

$$\Delta E_{ij}^{L} = \frac{1}{2}\left(\frac{\partial \Delta u_i}{\partial x_j} + \frac{\partial \Delta u_j}{\partial x_i} + \frac{\partial u_k{}^t}{\partial x_i}\frac{\partial \Delta u_k}{\partial x_j} + \frac{\partial \Delta u_k}{\partial x_i}\frac{\partial u_k{}^t}{\partial x_j}\right) \tag{36}$$

$$\Delta E_{ij}^{NL} = \frac{1}{2}\left(\frac{\partial \Delta u_i}{\partial x_i}\frac{\partial \Delta u_k}{\partial x_j}\right) \tag{37}$$

and Δu_i are the displacement increments from time t to t+Δt. Equation (34) becomes

$$\int_{V_o} (S_{ij}^{t} + \Delta S_{ij})\ \delta\ (\Delta E_{ij}^{L} + \Delta E_{ij}^{NL})\ dV = \delta W^{t+\Delta t} \tag{38}$$

Next assume the stress increment during any time step may be written

$$\Delta S_{ij} = C_{ijmn}^{t}\ (\Delta E_{mn} - \Delta E_{mn}^{C} - \Delta E_{mn}^{T}) + \Delta P_{ij} \tag{39}$$

where

C_{ijmn}^{t} = the effective modulus tensor at time t,

ΔE_{mn}^{C} = the creep strain increment from time t to t+Δt,

ΔE_{mn}^{T} = the thermal strain increment from time t to t+Δt, and

ΔP_{ij} = the stress increment due to creep and thermal loading.

It can be seen that the above constitutive relation is consistent with the material law presented in equation (29).

Now substitute equation (35) into (39) and (39) into equation (38) to obtain

$$\int_{V_o} S^t_{ij}\, \delta\Delta E^L_{ij}\, dV + \int_{V_o} S^t_{ij}\, \delta\Delta E^{NL}_{ij}\, dV + \int_{V_o} C^t_{ijmn}\, \Delta E^L_{mn}\, \delta\Delta E^L_{ij}\, dV$$

$$+ \int_{V_o} C^t_{ijmn}\, \Delta E^L_{mn}\, \delta\Delta E^{NL}_{ij}\, dV = \int_{V_o} C^t_{ijmn}\, \Delta E^{NL}_{mn}\, \delta\Delta E^L_{ij}\, dV$$

$$+ \int_{V_o} C^t_{ijmn}\, \Delta E^{NL}_{mn}\, \delta\Delta E^{NL}_{ij}\, dV - \int_{V_o} C^t_{ijmn}\, \Delta E^C_{mn}\, \delta\Delta E^L_{ij}\, dV$$

$$- \int_{V_o} C^t_{ijmn}\, \Delta E^C_{mn}\, \delta\Delta E^{NL}_{ij}\, dV - \int_{V_o} C^t_{ijmn}\, \Delta E^T_{mn}\, \delta\Delta E^L_{ij}\, dV$$

$$- \int_{V_o} C^t_{ijmn}\, \Delta E^T_{mn}\, \delta\Delta E^{NL}_{ij}\, dV + \int_{V_o} \Delta P_{ij}\, \delta\Delta E^L_{ij}\, dV$$

$$+ \int_{V_o} \Delta P_{ij}\, \delta\Delta E^{NL}_{ij}\, dV = \delta W^{t+\Delta t} \tag{40}$$

Terms 4,5 and 6 in equation (40) are nonlinear in the displacement increment. In order to allow for a linear incremental formulation, it will be assumed that the displacement increment is small so that these terms can be neglected at this time. However, this approximation will be accounted for later in the section describing equilibrium iteration. With this modification, the virtual work expression, with equation (30) included, simplifies to

$$
-\int_{V_o} \rho_o F_k \delta u_k \, dV + \int_{V_o} C^t_{ijmn} \, \Delta E^L_{mn} \, \delta\Delta E^L_{ij} \, dV + \int_{V_o} S^t_{ij} \, \delta\Delta E^{NL}_{ij} \, dV
$$

$$
-\int_{V_o} C^t_{ijmn} \, \Delta E^C_{mn} \, \delta\Delta E^{NL}_{ij} \, dV - \int_{V_o} C^t_{ijmn} \, \Delta E^T_{mn} \, \delta\Delta E^{NL}_{ij} \, dV
$$

$$
+\int_{V_o} \Delta P_{ij} \, \delta\Delta E^{NL}_{ij} \, dV = \int_{A} T_k \delta u_k dA - \int_{V_o} S^t_{ij} \, \delta\Delta E^L_{ij} \, dV + \int_{V_o} C^t_{ijmn} \, \Delta E^C_{mn} \, \delta\Delta E^L_{ij} \, dV
$$

$$
+\int_{V_o} C^t_{ijmn} \, \Delta E^T_{mn} \, \delta\Delta E^L_{ij} \, dV - \int_{V_o} \Delta P_{ij} \, \delta\Delta E^L_{ij} \, dV \tag{41}
$$

The above virtual work equation applies for elastic-plastic action as well as creep and thermal effects. Note that if thermal and creep effects are neglected equation (41) reduces to the virtual work expression presented by Hunsaker.

FINITE ELEMENT DISCRETIZATION

The process of obtaining a finite element approximation of the linearized virtual work expression [equation (41)] will now be summarized. For a more detailed description, the reader is referred to Hunsaker.

First recall that for an isoparametric element the displacements, u_i, and coordinates, x_i, of points in a body may be related via the element nodal displacements, u_i^α, and coordinates, x_i^α, by using shape functions, h^α.

$$
u_i = h^\alpha u_i^\alpha \tag{42}
$$

$$
x_i = h^\alpha u_i^\alpha \tag{43}
$$

Note that repeated Greek indices indicate summation over the total number of nodes in the element.

The strain increment may thus be related to the displacement by

$$\Delta E^{L}_{ij} = \frac{1}{2}\left(\frac{\partial h^{\alpha}}{\partial x_j}\,\Delta u^{\alpha}_{i} + \frac{\partial h^{\alpha}}{\partial x_i}\,\Delta u^{\alpha}_{j} + \frac{\partial h^{\alpha}}{\partial x_i}\frac{\partial h^{\beta}}{\partial x_j}\,u^{\alpha}_{k}\,\Delta u^{\beta}_{k} + \frac{\partial h^{\alpha}}{\partial x_i}\frac{\partial h^{\beta}}{\partial x_j}\,\Delta u^{\alpha}_{j}\,u^{\beta}_{k}\right) \tag{44}$$

$$\Delta E^{NL}_{ij} = \frac{1}{2}\frac{\partial h^{\alpha}}{\partial x_i}\frac{\partial h^{\beta}}{\partial x_j}\,\Delta u^{\alpha}_{k}\,\Delta u^{\beta}_{k} \tag{45}$$

The displacement and strain variations in equation (41) may now be written for a single element as

$$\delta\Delta E^{L}_{rs} = \frac{\partial\Delta E^{L}_{rs}}{\partial u^{\alpha}_{i}}\,\delta u^{\alpha}_{i} = \frac{\partial\Delta E^{L}_{rs}}{\partial\Delta u^{\alpha}_{i}}\,\delta u^{\alpha}_{i} \tag{46}$$

$$\delta\Delta E^{NL}_{ij} = \frac{\partial\Delta E^{NL}_{ij}}{\partial u^{\alpha}_{k}}\,\delta u^{\alpha}_{k} = \frac{\partial\Delta E^{NL}_{ij}}{\partial\Delta u^{\alpha}_{k}}\,\delta u^{\alpha}_{k} \tag{47}$$

$$\delta u_k = \frac{\partial u_k}{\partial u^{\alpha}_{i}}\,\delta u^{\alpha}_{i} = \frac{\partial\Delta u_k}{\partial\Delta u^{\alpha}_{i}}\,\delta u^{\alpha}_{i} \tag{48}$$

Now define $\{R^{t+\Delta t}\}$ by

$$R^{t+\Delta t}_{i} = \int_{A} T_k\,\frac{\partial\Delta u_k}{\partial\Delta u^{\alpha}_{i}}\,dA + \int_{V_o} \rho_o F_k\,\frac{\partial\Delta u_k}{\partial\Delta u^{\alpha}_{i}}\,dV \tag{49}$$

Next, in order to evaluate the second integral in equation (41), first define

$$\{\Delta S\} = [C]\,\{\Delta E\} \tag{50}$$

where

$$\{\Delta S\}^T = \lfloor\, \Delta S_{11}\ \Delta S_{22}\ \Delta S_{33}\ \Delta S_{12}\ \Delta S_{13}\ \Delta S_{23} \,\rfloor \tag{51}$$

and

$$\{\Delta E\} = \lfloor \Delta E_{11}\ \Delta E_{22}\ \Delta E_{33}\ \Delta E_{12}\ \Delta E_{13}\ \Delta E_{23} \rfloor \tag{52}$$

and [C] is the matrix representation of C_{ijmn} written in engineering strain notation .

Also, write

$$\{\Delta E_L\} = [B_L]\ \{\Delta u_\alpha\} \tag{53}$$

where [B_L] is the linear strain displacement transformation matrix obtained from equation (44). Thus, when applied at all nodes in the element, the second term in the virtual work expression may be written

$$\sum_{k=1}^{3} \sum_{\alpha=1}^{n} \int_{V_o} C_{ijrs}\ \Delta E_{ij}^{L}\ \frac{\partial \Delta E_{rs}}{\partial u_k^\alpha}\ dV = [K_L]\ \{\Delta u_\alpha\} \tag{54}$$

where n is the number of nodes in the element and

$$[K_L] = \int_{V_o} [B_L]^T\ [C]\ [B_L]\ dV \tag{55}$$

The remaining terms on the left hand side of equation (41) are found in a similar manner to yield

$$\sum_{k=1}^{3} \sum_{\alpha=1}^{n} \left[\int_{V_o} S_{ij}^{t}\ \frac{\partial E_{ij}^{NL}}{\partial \Delta u_k^\alpha}\ dV - \int_{V_o} C_{ijmn}^{t}\ \Delta E_{mn}^{C}\ \frac{\partial E_{ij}^{NL}}{\partial \Delta u_k^\alpha}\ dV \right.$$

$$\left. - \int_{V_o} C_{ijmn}^{t}\ \Delta E_{mn}^{T}\ \frac{\partial E_{ij}^{NL}}{\partial \Delta u_k^\alpha}\ dV + \int_{V_o} \Delta P_{ij}\ \frac{\partial E_{ij}^{NL}}{\partial \Delta u_k^\alpha}\ dV \right] = [K_{NL}]\ \{\Delta u_\alpha\} \tag{56}$$

where

$$[K_{NL}] = \int_{V_o} [B_{NL}]^T ([S] - [C][\Delta E^C] - [C][\Delta E^T] + [\Delta P]) [B_{NL}]\, dV \tag{57}$$

and $[B_{NL}]$ is the nonlinear strain-displacement transformation matrix obtained from equation (45).

The remaining four terms in equation (41) may be written

$$\sum_{k=1}^{3}\sum_{\alpha=1}^{n}\left[-\int_{V_o} S_{ij}^{t} \frac{\partial \Delta E_{ij}^{L}}{\partial \Delta u_k^{\alpha}}\, dV + \int_{V_o} C_{ijmn}^{t}\, \Delta E_{mn}^{C} \frac{\partial \Delta E_{ij}^{L}}{\partial \Delta u_k}\, dV \right.$$

$$\left. + \int_{V_o} C_{ijmn}^{t}\, \Delta E_{mn}^{T} \frac{\partial \Delta E_{ij}^{L}}{\partial \Delta u_k^{\alpha}}\, dV - \int_{V_o} \Delta P_{ij} \frac{\partial \Delta E_{ij}^{L}}{\partial \Delta u_k^{\alpha}}\, dV \right] =$$

$$\int_{V_o} [B_L]^T (\{S\} - [C]\{\Delta E^C\} - [C]\{\Delta E^T\} + \{\Delta P\})\, dV = -\{F^t\} \tag{58}$$

If the body force term in equation (41) includes inertial effects, the following term may be separated out and brought to the left hand side:

$$\int_{V_o} \rho_o \ddot{u}_k^{t+\Delta t} \frac{\partial \Delta u_k}{\partial \Delta u_i^{\alpha}}\, dV = [M]\{\ddot{u}_\alpha^{t+\Delta t}\} \tag{59}$$

where [M] is the consistent mass matrix.

The global equations of motion for the entire structure may thus be assembled from the element equations to obtain

$$[M]\{\ddot{u}_\alpha^{t+\Delta t}\} + ([K_L^t] + [K_{NL}^t])\{\Delta u_\alpha\} = \{R^{t+\Delta t}\} - \{F^t\} \tag{60}$$

The solution of the above equations will result in approximate displacement increments due to the neglection of nonlinear terms in the virtual

work expressions. This approximation may be accounted for by performing equilibrium iteration.

EQUILIBRIUM ITERATION

Since equation (41) is not exact, it is necessary to perform equilibrium iteration on each load step to obtain the exact displacements. This is accomplished by utilizing the virtual work equation in incremental form [equation (33)] and recognizing that for exact equilibrium

$$\{R^{t+\Delta t}\} = \{F^{t+\Delta t}\} \tag{61}$$

where $\{R^{t+\Delta t}\}$ is the internal force term defined by equation (49), and $\{F^{t+\Delta t}\}$ is the external force term defined by

$$\{F^{t+\Delta t}\} = \int_{V_o} [B_L{}^{t+\Delta t}]^T \{S^{t+\Delta t}\}\, dV \tag{62}$$

For dynamic problems, the inertia effect may be included in equation (62). To improve the initial estimate one can define

$$\{f\} = -\{F^{t+\Delta t}\} + \{R^{t+\Delta t}\} \tag{63}$$

so that for exact equilibrium $\{f\} = \{0\}$. Applying the modified Newton method to solve the nonlinear system of equations given by (61), one obtains

$$([K_L{}^t] + [K_{NL}^t])\{\Delta\Delta q_i\} = -\{F^{t+\Delta t}\}_i + \{R^{t+\Delta t}\}_i \tag{64}$$

$$q_{i+i}^{t+\Delta t} = q_i^{t+\Delta t} + \Delta\Delta q_i \tag{65}$$

Notice that $\{F^{t+\Delta t}\}$ is defined by equation (62) and not by equation (58). It should be noted that a strict application of the Newton method would require the use of $[K_L{}^{t+\Delta t}]$ and $[K_{NL}^{t+\Delta t}]$. However, the use of $[K_L^t]$ and $[K_{NL}^t]$ generally is sufficient and provides a computationally faster solution. If convergence is slow, the stiffness terms may be updated, i.e., use $[K_L^{t+\Delta t}]$ and $[K_{NL}^{t+\Delta t}]$.

SPECIAL LOADING CONDITIONS

The equations of motion described by equation (60) represent an elastic-plastic-creep-thermal load response for a material with temperature dependent material properties. Under certain special loading conditions the constitutive law and the equations of motion may be simplified so that the solution scheme may be computationally improved. The special cases considered herein are quasi-isothermal elastic action, nonisothermal elastic action, and quasi-isothermal elastic-plastic action, where quasi-isothermal action is defined to be any load response which includes thermal loads but for which the material properties may be considered constant over the temperature range encountered during the load history. The three loading conditions above, together with the previously derived nonisothermal elastic-plastic action, are chosen because of their diverse programmabilities into the computer code AGGIE I.

Quasi-Isothermal Elastic Action

During quasi-isothermal elastic action the constitutive relation given by equation (39) may be simplified to

$$\Delta S_{ij} = D^t_{ijmn} \left(\Delta E_{mn} - \Delta E^C_{mn} - \Delta E_{mn}{}^T\right) \tag{66}$$

where $D_{ijmn}{}^T$ is the elastic constitutive tensor at time t. For this case the linearized virtual work expression is

$$- \int_{V_o} \rho_o F_k \, \delta u_k \, dV + \int_{V_o} D^t_{ijmn} \, \Delta E^L_{mn} \, \delta\Delta E^L_{ij} \, dV$$

$$+ \int_{V_o} (S^t_{ij} - D^t_{ijmn} \, \Delta E^C_{mn} - D^t_{ijmn} \, \Delta E_{mn}{}^T) \, \delta\Delta E^{NL}_{ij} \, dV =$$

$$\int_A T_k \delta u_k \, dA - \int_{V_o} (S^t_{ij} - D^t_{ijmn} \, \Delta E^C_{mn} - D^t_{ijmn} \, \Delta E_{mn}{}^T) \, \delta\Delta E^L_{ij} \, dV \tag{67}$$

If certain terms are defined in accordance with the above equation, the equations of motion given in equation (60) need not be altered. For the loading case specified here the newly defined terms are

$$[K_{NL}] = \int_{V_o} [B_{NL}]^T \left([S] - [D]\,[\Delta E^C] - [D]\,[\Delta E^T]\right) [B_{NL}]\, dV \tag{68}$$

and

$$\{F^t\} = \int_{V_o} [B_L]^T \left(\{S\} - [D]\,\{\Delta E^C\} - [D]\,\{\Delta E^T\}\right) dV \tag{69}$$

where [D] is the elastic constitutive matrix. In addition, [S] and {S} are the stress matrix and stress vector, respectively, described by Hunsaker, and [E] and {E} are similar strain increment representations.

Nonisothermal Elastic Action

When the load response is nonisothermal elastic the stress at time t is

$$S^t_{ij} = D^t_{ijmn} \left({E_{mn}}^t - {E_{mn}}^{Ct} - {E_{mn}}^{Tt}\right) \tag{70}$$

The total increment from equation (13) is

$$\begin{aligned} \Delta S_{ij} &= D^{t+\Delta t}_{ijmn} \left(\Delta E_{mn} - \Delta {E_m}^C - \Delta {E_{mn}}^T\right) \\ &\quad + \Delta D_{ijmn} \left({E_{mn}}^t - {E_{mn}}^{Ct} - {E_{mn}}^{Tt}\right) \end{aligned} \tag{71}$$

where ΔD_{ijmn} represents the change in the elastic constitutive tensor during a time step due to a temperature change. Note that the entire last term represents ΔP_{ij} in equation (41). Since D_{ijmn} can be determined directly from input stress-strain data, the above constitutive relation may be substituted into the virtual work expression which becomes

$$- \int_{V_o} \rho_o F_k \delta u_k \, dV + \int_{V_o} D^{t+\Delta t}_{ijmn} \Delta E^L_{mn} \quad \delta \Delta E^L_{ij} \quad dV$$

$$+ \int_{V_o} [S^t_{ij} - D^{t+\Delta t}_{ijmn} \Delta E_{mn}{}^C - D^{t+\Delta t}_{ijmn} \Delta E_{mn}{}^T + \Delta D_{ijmn} (E^t_{mn} - E_{mn}{}^{Ct}$$

$$- E_{mn}{}^{Tt})] \; \delta \Delta E^{NL}_{ij} \; dN = \int_A T_k \delta u_k \, dA - \int_{V_o} [S^t_{ij} - D^{t+\Delta t}_{ijmn} \Delta E_{mn}{}^C - D^{t+\Delta t}_{ijmn} \Delta E_{mn}{}^T$$

$$- \Delta D_{ijmn} (E^t_{mn} - E_{mn}{}^{Ct} - E_{mn}{}^{Tt})] \; \delta \Delta E^L_{ij} \; dV \tag{72}$$

Once again the discretization process yields equation (60) where now

$$[K_{NL}] = \int_{V_o} [B_{NL}]^T ([S] - [D] [\Delta E^C] - [D] [\Delta E^T]$$

$$+ [\Delta D] [E^t - E^{Ct} - E^{Tt}]) [B_{NL}] \, dV \tag{73}$$

and

$$\{F^t\} = \int_{V_o} [B_L]^T (\{S\} - [D] \{\Delta E^C\} - [D] \{\Delta E^T\}$$

$$+ [\Delta D] \{E^t - E^{Ct} - E^{Tt}\}) \, dV \tag{74}$$

Quasi-Isothermal Elastic-Plastic Action

When the load response is such that the thermal effects may be considered quasi-isothermal, the elastic-plastic constitutive relation reduces to

$$\Delta S_{ij} = C^t_{ijmn} (\Delta E_{mn} - \Delta E_{mn}{}^C - \Delta E_{mn}{}^T) \tag{75}$$

With this simplification the virtual work equation may be written

$$- \int_{V_o} \rho_o F_k \delta u_k \, dV + \int_{V_o} C^t_{ijmn} \, \Delta E^L_{mn} \, \delta \Delta E^L_{ij} \, dV$$

$$+ \int_{V_o} (S^t_{ij} - C^t_{ijmn} \, \Delta E_{mn}{}^C - C^t_{ijmn} \, \Delta E_{mn}{}^T) \, \delta \Delta E^{NL}_{ij} \, dV =$$

$$\int_A T_k \delta u_k \, dA - \int_{V_o} (S^t_{ij} - C^t_{ijmn} \, \Delta E_{mn}{}^C - C^t_{ijmn} \, \Delta E_{mn}{}^T) \, \delta \Delta E^L_{ij} \, dV \tag{76}$$

defining

$$[K_{NL}] = \int_{V_o} [B_{NL}]^T ([S] - [C] [\Delta E^C] - [C] [\Delta E^T]) [B_{NL}] \, dV \tag{77}$$

and

$$\{F^t\} = \int_{V_o} [B_L]^T (\{S\} - [C] \{\Delta E^C\} - [C] \{\Delta E^T\}) \, dV \tag{78}$$

yields the equations of motion given by equation (60).

It should be noted here that an analysis similar to that performed in the section dealing with the general constitutive law will show that the constitutive tensor for this loading case is identical to that obtained in equation (21).

CONCLUSION

It has been shown by Haisler[3] that for cyclic load histories neither isotropic nor kinematic hardening models accurately predict the Bauschinger effect for many materials. It has been shown further that the combined isotropic-kinematic hardening rule more properly depicts material response when isothermal conditions are encountered. A material model which accounts

for the Bauschinger effect and includes both creep and thermal response has been proposed in this paper. It is believed by these authors that this model will more accurately predict material response than a kinematic hardening rule when thermal loads are present.

The proposed model presented here is currently being studied in the finite element code AGGIE I. Results of this study are forthcoming.

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In this paper, a formulation is presented for a combined isotropic-kinematic hardening rule which includes both creep and thermal effects. The resulting model accounts for temperature dependent material properties as well as the Bauschinger effect for reverse loading, the virtual work equation and resulting matrix equations are derived for implementation of the model into a general purpose finite element code.